It would appear from the foregoing that the same is also true for annuli.

From this work it is apparent that the mixing length constant is not in fact a universal constant for all surfaces, as postulated by some workers, but is in fact a function of the radius or curvature ratio of the annulus to which it applies.

The method contained herein appears to give a simple, but reliable, prediction of the form of the $u^+ - y^+$ relationship for the inner region of annuli.

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ASYMMETRIC HEAT TRANSFER IN TURBULENT FLOW BETWEEN PARALLEL PLATES

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NOMENCLATURE

D_e, = 2s, equivalent diameter of the channel;
f, function;
h, convective heat-transfer coefficient;
Nu, Nusselt number (based on equivalent diameter);
Re, Reynolds number (based on equivalent diameter);
s, width of the channel;
T, temperature;
u, velocity;
q heat flux;
x, distance in the direction of flow;
y, distance from a wall

Greek symbols
α, dimensionless temperature as defined in equations (11) and (12);
γ, $= (q''_w/q'_w)$, ratio of the heat fluxes at the walls of the channel;
ϵ, total conductivity of heat.

Subscripts
PS, value in symmetrical heating case;
P, Q, refer to the walls of the channel;
w, wall value;
B, bulk value.

Superscripts
$', '', '''$, refer to the thermal boundary conditions described in Fig. 1.

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INTRODUCTION

A NUMBER of analytical studies [1–5] have been made of heat-transfer coefficients in flow between parallel plates with unequal heat fluxes at the two plates. An experiment has been reported by Barrow [4], but the results showed considerable scatter and its was difficult to confirm that the analysis given by Barrow in the same paper adequately described the variation in the heat-transfer coefficient (at one wall) with the ratio of the fluxes at the two walls, γ.

In most of the previous work, an analytical solution is first obtained for flow between the plates with heat transfer at one wall and the other wall insulated. Assumptions for the variations of the velocity and eddy diffusivity of heat across the channel are made. The case of asymmetric heat transfer at both walls is dealt with by adding together the solutions for the temperature when heat is transferred at each wall in turn with the other wall insulated. The resulting heat-transfer coefficients at the two walls are shown to be functions of the heat flux ratio, γ, i.e. the ratio of the heat transfer at one wall to the heat transfer at the other wall.

This method of superposition proposed by Seban [1] Stein [2] and Hatton and Quarmby [5] is briefly reviewed here, but no assumptions for the velocity and eddy diffusivity distributions are made. The new point established is that for fully developed flow, a simple relation for the variation of heat-transfer coefficient (at either wall) with heat flux ratio may be derived if the heat-transfer coefficients for the insulated wall case and for the case of equal and opposite heat fluxes are known, independent of the detailed assumptions concerning the velocity and eddy diffusivity distributions. This result also follows from Hatton and Quarmby's work [5] [equation (24) and (25) of their paper] if fully developed flow is considered.

A favourable comparison is made between heat-transfer coefficients predicted from this superposition analysis, and experimental data reported by Barrow [4].

ANALYSIS

Consider first the heat transfer from one wall P to a turbulent flow between plates P and Q as shown in Fig. 1(a). The wall Q is insulated. The heat flux per unit area at the wall P is q'_w and is invariant with x, the distance along the channel. Wall temperatures are T'_P and T'_Q and the bulk mean temperature of the fluid is T'_B.

The heat-transfer coefficient is, by definition

$$h'_P = q'_w/(T'_P - T'_B) \tag{1}$$

and the temperature distribution across the channel may be expressed non-dimensionally as

$$\frac{T' - T'_Q}{T'_P - T'_Q} = f'(y). \tag{2}$$

Consider next the case when the wall Q is heated and the wall P is insulated, Fig. 1(b). The heat transfer at the wall Q is now $q''_w = \gamma q'_w$, and is invariant with x, so that γ is constant. The heat-transfer coefficient is

$$h''_Q = \frac{q''_w}{T''_Q - T''_B} = \frac{\gamma q'_w}{T''_Q - T''_B} \tag{3}$$

where T''_Q and T''_B are respectively the heated wall temperature and the bulk mean temperature, and the temperature distribution may be expressed as

$$\frac{T'' - T''_P}{T''_Q - T''_P} = f''(s - y) \tag{4}$$

where s is the width of the channel.

It is now assumed that the distribution of the total conductivity of heat ϵ is symmetrical across the channel and

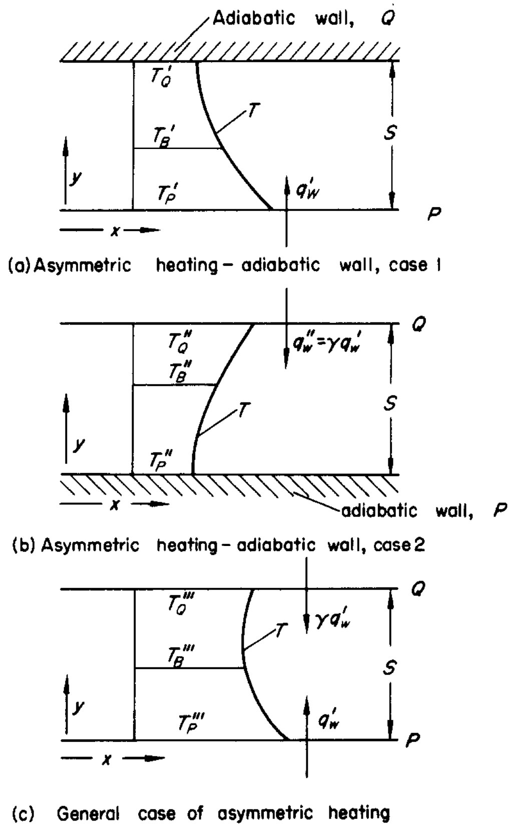

FIG. 1. Temperature distributions in asymmetric heating.

is the same in the two cases, i.e. $\epsilon' = \epsilon'' = \epsilon(y)$. The velocity distribution will also be the same, i.e. $u' = u'' = u(y)$. The energy equation is linear in each case, the simplified forms of the equations being

$$u\frac{\partial T'}{\partial x} = \frac{\partial}{\partial y}\left(\epsilon\frac{\partial T'}{\partial y}\right) \tag{5}$$

and

$$u\frac{\partial T''}{\partial x} = \frac{\partial}{\partial y}\left(\epsilon\frac{\partial T''}{\partial y}\right) \tag{6}$$

and the temperature functions are identical, i.e. $f' \equiv f''$. The energy equations may be added to give

$$u\frac{\partial}{\partial x}(T' + T'') = \frac{\partial}{\partial y}\left(\epsilon\frac{\partial}{\partial y}(T' + T'')\right). \tag{7}$$

This equation then describes the flow when there is heat transfer from both walls simultaneously as illustrated in Fig. 1(c). The local temperature distribution for the general case is given by

$$T''' = T' + T'' = f(y)(T'_P - T'_Q) + f(s-y)(T''_Q - T''_P) + T'_Q + T''_P \quad (8)$$

or,

$$\frac{T''' - T'''_Q}{T'''_P - T'''_Q} = \frac{f(y) + \gamma[f(s-y) - 1]}{1-\gamma} \quad (9)$$

where

$$T'''_P = T'_P + T''_P, \quad T'''_Q = T'_Q + T''_Q$$

and

$$\gamma = \frac{T''_Q - T''_P}{T'_P - T'_Q} = \frac{q''_w}{q'_w}.$$

Thus the temperature distribution with unequal heat transfer at the two walls may be described in terms of the complete solution of the adiabatic wall case, $f(y)$, and the ratio of the heat fluxes, γ. The heat-transfer coefficient at the wall P is

$$h'''_P = \frac{q'_w}{T'''_P - T'''_B} = \frac{q'_w}{(T'_P - T'_B) + (T''_P - T''_B)}$$

$$= \frac{1}{\left(\frac{T'_P - T'_B}{q'_w}\right) + \left(\frac{T''_Q - T''_B}{q'_w}\right) + \left(\frac{T''_P - T''_Q}{q'_w}\right)}$$

$$= \frac{1}{\frac{1}{h'_P} + \frac{\gamma}{h''_Q} + \left(\frac{T''_P - T''_Q}{q'_w}\right)}. \quad (10)$$

But, $h'_P = h''_Q$ and $q'_w = h'_P(T''_Q - T''_B)/\gamma$ from equation (3) so that,

$$\frac{h'''_P}{h'_P} = \frac{1}{1 + \gamma + \gamma\left[\frac{T''_P - T''_Q}{T''_Q - T''_B}\right]}$$

$$= \frac{1}{1 + \gamma(1-\alpha)} \quad (11)$$

where,

$$\alpha = \left(\frac{T''_Q - T''_P}{T''_Q - T''_B}\right).$$

Similarly,

$$\frac{h'''_Q}{h'_P} = \frac{1}{1 + (1-\alpha)/\gamma} \quad (12)$$

where

$$\alpha = \left(\frac{T'_P - T'_Q}{T'_P - T'_B}\right).$$

EXPERIMENTAL INTERPRETATION

It follows from the analysis, that if the heat transfer coefficients for the adiabatic wall case ($\gamma = 0$) and for symmetrical heating ($\gamma = 1$) are known, then the heat-transfer coefficient for any other value of γ may be determined. It should be noted, after Seban (1), that the heat-transfer coefficient changes with γ because of the change in the bulk mean temperature—the non-dimensional temperature gradient at one wall, viz. $[\partial f'(y)/\partial y]_{y=0}$ is unchanged by the heating at the other wall.

Considering the wall P, it follows from equation (11) that if $\gamma = 0$

$$h'''_{P(\gamma=0)} = h'_P$$

as required by the definition of h'_P.

If $\gamma = 1$, the case uf symmetrical heating, then from equation (11),

$$\frac{h_{PS}}{h'_P} = \frac{1}{2-\alpha} \quad (13)$$

where h_{PS} is the heat-transfer coefficient at the wall P in symmetrical heating. From equations (11) and (13)

$$\frac{h'''_P}{h'_P} = \frac{Nu'''_P}{Nu'_P} = \frac{1}{1 - \gamma(1 - h'_P/h_{PS})} \quad (14)$$

Similarly, for the wall Q

$$\frac{h'''_Q}{h'_P} = \frac{Nu'''_Q}{Nu''_Q} = \frac{1}{1 + (h'_P/h_{PS} - 1)/\gamma} \quad (15)$$

It should be noted here that equations [14] and [15] are compatible with the results of Hatton and Quarmby's analysis, i.e. their equations [24] and [25] for large axial distance.

Barrow [4] has carried out an experiment in which the heat-transfer coefficient h'_P, and h'''_P for various negative values of γ, were measured. In his parallel walled duct, the most reliable results were obtained in the central region $37{\cdot}5 < x/D_e < 75$ where $D_e (= 2s)$ is the equivalent diameter of the duct. Measurements of the heat-transfer coefficient in symmetrical heating were not made and so h'''_P was related to h'_P.

In a series of adiabatic wall experiments which were conducted over a range of Reynolds number (11000–26000) it was found that at $x/D_e \doteq 50$ the Nusselt numbers were on the average 0·888 that value for symmetrical heating, i.e. $(h'_P/h_{PS}) = 0{\cdot}888$. For symmetrical heating Nu_{PS} was evaluated from a theoretical result which agreed favourably with $Nu = 0{\cdot}02\, Re^{0{\cdot}8}$ where Re is the Reynolds number based on the equivalent diameter D_e. According to equation (11) therefore,

$$\frac{Nu'''_P}{Nu'_P} = \left[\frac{1}{1 - 0{\cdot}112\,\gamma}\right] \quad (16)$$

This relation is shown in Fig. 2 together with the experimental data for asymmetrical heating obtained by Barrow [4] at the same axial location.

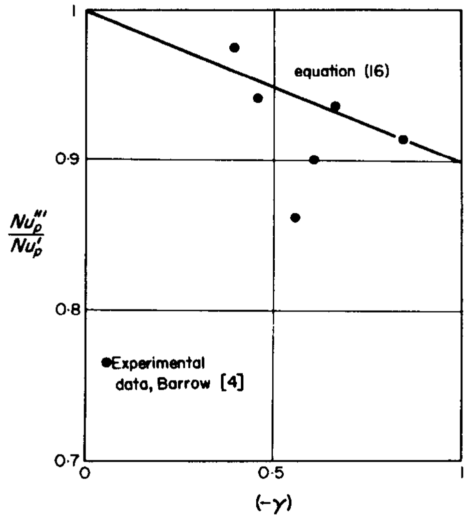

FIG. 2. Dependence of heat-transfer coefficient on asymmetry of heat transfer.

CONCLUSIONS

Following Seban [1] and Stein [2], it is shown that if the temperature distribution across a channel, heated at one wall and insulated at the other, is completely determined (experimentally or analytically) then the temperature distribution in a more general asymmetric case, of unequal heating at the two walls, may be derived.

From an experimental point of view it may be difficult to obtain temperature distributions. Nevertheless, if heat-transfer coefficients for the cases of one adiabatic wall and symmetrical heating are obtained experimentally, then the heat-transfer coefficients at the two walls, in the general asymmetric case, are easily obtained from equations (11) and (12).

Fair confirmation of these relations is obtained from experimental data already published by Barrow [4].

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